Grip Vs Lean Angle

This article explains how tyre grip varies as a motorcycle leans off the vertical, when taking a bend.

There are two parts to the article:

- **Part A:** This gives the non-technical description of what happens to grip when the bike leans;

- **Part B:** This gives a deep technical description of what happens and why it happens.

Those interested in 'what happens' should read Part A and those who want to know 'why' it happens are invited to look at Part B.

Part A: Grip Vs Lean

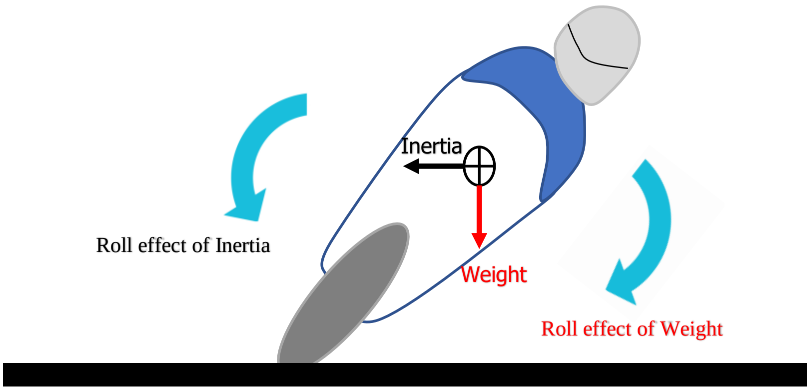
The feel of a bike leaning over in a bend is awesome and probably one of the main reasons why we ride bikes: it's great fun!

However, have you ever stopped to consider how a bike can lean over so far without falling on its side? Clearly it has something to do with speed, because if you lean your bike over when it is at rest and let it go….well, you know what happens!

It must also have a lot to do with both the tyres and the road surface; we all know that you cannot lean over as far on a slippery surface; whereas race bikes can lean to about 60^0 on a track, because of the super 'grip' provided by a race track surface and race tyres!

To find out 'why' a bike can lean without falling over we need to turn to the language of nature: mathematics! However, we can cut to chase and simply state why things happen and that's the purpose of this section of the article on 'Grip Vs Lean'.

The reason a bike balances in a bend at speed is because of two opposing forces, one of which -the Weight of the bike and rider- is trying to make it roll inwards, towards the centre of the bend; and the other – the bike's Inertia- is trying to make it roll outwards, away from the centre of the bend.

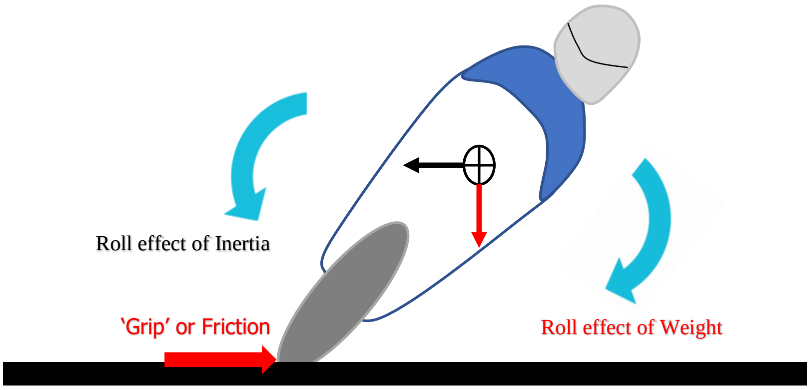


On entering a bend, lean angle increases until the 'roll effect' of these opposing forces (Weight and Inertia) balance each other out.

Look at the diagram below and ask yourself the following question: *'why do the bike tyres not slip out to the left, (as you look at the picture), away from the centre of the bend?'*

It's a good question and although the tyres are trying to slip out to the left, they are prevented from doing so because of Friction, between the tyres and the road surface.

The size of this Friction Force depends indirectly on the Weight of the bike and rider; and it depends directly on the material from which the tyres are made and the construction of the road surface. Together these factors combine and determine how big the Friction Force will be.



If the bike were cornering on a slippery surface, such as ice, the Friction Force would be much smaller, and the bike would be more likely to drop on its side. Likewise, if the tyres were made of wood instead of rubber, the Friction Force would be much smaller, and the bike would not be able to lean in the way it does with nice grippy rubber tyres!

The key question we set out to answer, is how does the lean angle affect grip?

Another name for 'grip' is Friction, so we are really asking what happens to this Friction Force as the bike leans over.

The answer is that it gets bigger! The more the bike leans off the vertical, the bigger the Friction Force becomes.

At this point you are probably wondering why it is that bikes can fall over at all, if the Friction Force gets bigger the more they lean!

It's a good point and in 'theory' if the lean angle was 90^0, which would be the case if the bike was horizontal and lying in the same plane as the road surface, then the Friction Force would have to be infinite!

Clearly no tyre has 'infinite' grip, so what happens in real life is that the Friction Force (I.E. the 'grip') increases but only up to some limiting value that is determined by the nature of the tyre; the nature of the road surface; the weight of the bike and rider and the lean angle.

Once this limit is reached, the tyre 'lets go' and the contact patch, where the tyre is in contact with the road, simply loses adhesion; slides away from the centre of the bend and the bike goes down! In summary and contrary to what some bikers believe, 'grip' increases with lean angle; but only up to a point!

If you want to explore the physics associated with grip and lean angle, please read Part B of this article, which is shown below.

George A Cairns

Part B: Grip Vs Lean

The technical material used in this article is based on information extracted from 'Motorcycle Dynamics' by Professor Vittorre Cossalter. Although much of the physics involved also applies to cars, the article is mainly about the 'grip' generated when a motorcycle leans into a bend.

Throughout his book, Professor Cossalter refers to Centrifugal Force and not Centripetal Force, because he uses Non-Inertial Frames of Reference for measurements. (I.E. X-Y-Z Co-ordinate Axes assumed to be attached to the bike). When using a Non-Inertial Frame of Reference, it is correct to show Centrifugal Force in calculations involving the forces acting on a bike in a bend.

Grip:

Let us start by defining what is meant by the term 'grip', with reference to motorcycle tyres.

'Grip' is another name for friction and friction is a force that is generated when one surface moves over another with which it is in contact. All forces are ***vector*** quantities which means that they have both ***size*** and ***direction*** components. When vectors are added together, both the size and the direction are considered.

'Grip' is therefore the vector sum of two Frictional Forces:

- **Lateral Friction**, which acts on the contact patches at right angles to the motorcycle's direction of travel, opposing the tendency of the wheels to slip sideways as the bike leans into a bend;

- **Longitudinal Friction**, which opposes both the Driving Force and the Braking Force, either of which will apply, depending on what the rider demands from the bike.

The summation of these Friction Forces is usually shown on a Friction Ellipse:

(Reference: Motorcycle Dynamics Pages 64 – 65)

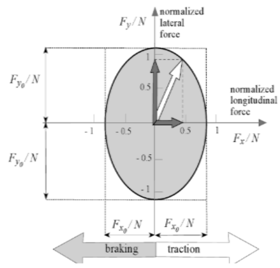
The diagram on the right shows how the Lateral and Longitudinal Friction Force vectors are added together to give the combined total Friction Force acting on the motorcycle: I.E. the combined 'grip' from both Lateral and Longitudinal friction components.

The vector sum of the Lateral and Longitudinal Friction Forces must be within the Friction Ellipse that has maximum values of F_{x_0} Longitudinal and F_{y_0} Lateral respectively, when they act alone.

$$F_{x_0} = \mu_{x_p} N \quad \text{and} \quad F_{y_0} = \mu_{y_p} N$$

2.7 Combined lateral and longitudinal forces: the friction ellipse P64

The longitudinal force F_x, either driving (positive value of F_x) or braking (negative value of F_x), is assumed to be assigned since it is controlled by the rider. The lateral force F_y that can be exercised is reduced by the simultaneous presence of the longitudinal force.

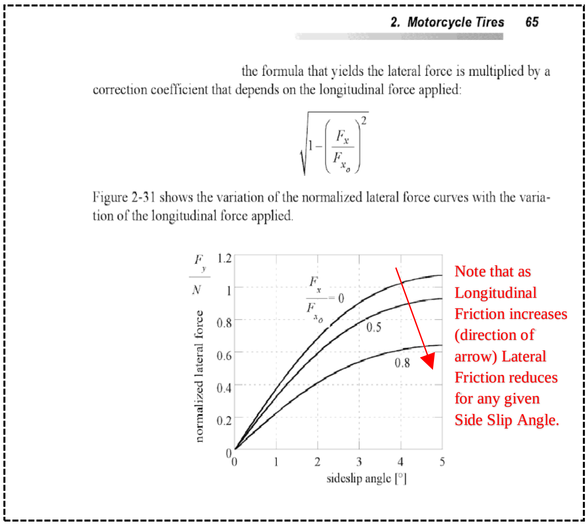


Where μ_{x_p} is the Longitudinal Traction Coefficient and μ_{y_p} is the Lateral Traction Coefficient. Professor Cossalter uses the term ***Normalised Lateral Force***, which is simply the Lateral Force divided by the Normal Load (N); this has two advantages, first the measured quantity is non-dimensional and second when either Friction Force equals the Normal Load, the 'Normalised' value becomes one; which simplifies the graphs.

Whether Lateral or Longitudinal: $Friction = \mu N$; where μ (Greek letter Mu) is the Traction Coefficient (also referred to as the Coefficient of Friction): a dimensionless quantity determined by the nature of the two sliding surfaces in contact and typically ranging in value from 0 - 1.4 The higher the value of μ, the bigger the 'grip'.

Newton's Third Law states that every action force, has an equal and opposite reaction force.

N is the Newton's Third Law equal and opposite force to the weight of the bike and rider, referred to as the Normal Reaction Force or the Normal Load. The word 'equal' tells us the size of N is the same size as the Weight of the bike and rider; and the word 'opposite' tells us that the Normal Load acts ***upwards*** on the tyres at the front and rear contact patches; which is the 'opposite' direction to the Weight, which acts downwards and is generally shown for mathematical convenience to act through the Centre of Gravity; which is also the Centre of Mass for fairly compact bodies such as motorcycles.


***2. Motorcycle Tires* 65**

the formula that yields the lateral force is multiplied by a correction coefficient that depends on the longitudinal force applied:

$$\sqrt{1-\left(\frac{F_x}{F_{x_o}}\right)^2}$$

Figure 2-31 shows the variation of the normalized lateral force curves with the variation of the longitudinal force applied.

The Friction Ellipse needs a little bit of explanation; for example, on the diagram on the previous page it states:

'The lateral Force F_y that can be exercised is reduced by the simultaneous presence of the Longitudinal Force'.

What Professor Cossalter is saying here is that the maximum value of Lateral Friction Force allowed, ***reduces*** as the Longitudinal Friction Force ***increases***. This must be the case as the vector sum of the two must never exceed the Maximum Total Friction Force defined by the outer edge of the Friction Ellipse.

To help you visualise this, look at the Ellipse and imagine the Horizontal Arrow (Longitudinal Friction Force vector) getting bigger as the motorcycle accelerates and demands traction; as it does, the Vertical Arrow (Later Friction Force vector) must get smaller, so that their sum (the sloping arrow) is never bigger than the maximum value allowed, as shown by the edge of the Ellipse!

On Page 65, Professor Cossalter states that the Lateral Friction Force is reduced by a 'Coefficient' (a dimensionless number less than 1), whenever there is Longitudinal Friction present; that is, whenever the bike is moving, be it under Acceleration; Steady State Speed or Braking.

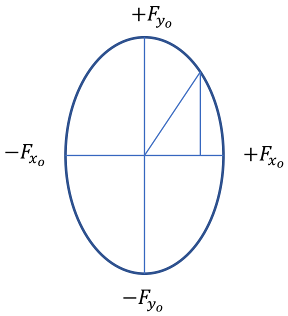
He then quotes the size of the 'coefficient' as $\sqrt{1 - \left(\frac{F_x}{F_{x_O}}\right)^2}$

In other words, the Normalised Lateral Friction: $F_y = \sqrt{1 - \left(\frac{F_x}{F_{x_O}}\right)^2}\ (\mu_{y_p} N)$

This value for Normalised Lateral Friction is smaller than the maximum value $F_y = \mu_{y_p} N$

as the Coefficient $\sqrt{1 - \left(\frac{F_x}{F_{x_o}}\right)^2}$ is less than one.

Hence the Lateral Friction Force is reduced from its maximum value of $\mu_{y_p} N$ by the Coefficient.



Note: Professor Cossalter does not show how this equation for the reduction Coefficient is derived, but I have done so below, starting from the equation for an Ellipse:

$$\frac{F_x^2}{F_{x_o}^2} + \frac{F_y^2}{F_{y_o}^2} = 1$$

Standard equation describing an Ellipse, modified to fit the Friction Ellipse used by Professor Cossalter.

$$\frac{F_y^2}{F_{y_o}^2} = 1 - \left(\frac{F_x}{F_{x_0}}\right)^2$$

$$F_y = F_{y_0} \sqrt{1 - \left(\frac{F_x}{F_{X_0}}\right)^2}$$

But as: $F_{y_0} = \mu_{y_p} N$

$$F_y = \mu_{y_p} N \sqrt{1 - \left(\frac{F_x}{F_{X_o}}\right)^2}$$

I.E. the maximum value of Lateral Friction F_{y_o} is reduced by the Coefficient, which is in turn determined by the Longitudinal Friction.

In summary, 'grip' is the vector sum of Longitudinal Friction, which acts in line with the direction of travel along the road plane and is generated by the forward motion of the bike; and Lateral Friction, which acts along the road plane at right angles to the bike's direction of travel and is generated when it leans off the vertical in a bend.

Does Weight affect 'grip'?

Weight always acts vertically down and in accordance with Newton's Second Law is the product of the Mass (m) of an object and the local Acceleration of Gravity (g): which is generally assumed to be 9.8 ms^{-2} at sea level.

$$W = mg$$

The question is: 'does Weight affect 'grip'?

The short answer is: 'yes it does, ***but indirectly,*** because of the relationship between Weight and the Normal Reaction Force'. Professor Cossalter refers to the Normal Reaction Force as the Normal Load and I shall use this term from now on.

Note: This diagram is explained on Pages 8 & 9 below.

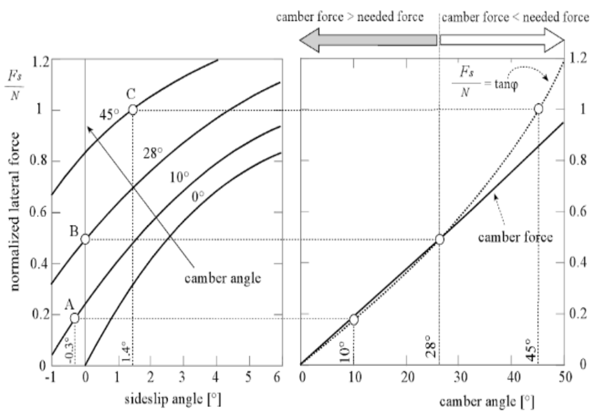


Fig. 2-18 Components of the lateral force generated by camber and slip. Tire A.

The Normal Load on the front tyre (N_f) and that on the rear tyre (N_r) when added together will equal the Weight when the motorcycle is at rest, standing on its wheels.

When the motorcycle is moving, the Aerodynamic Lift Force acting on the Centre of Pressure will act upwards and offset some of the Weight.

However, Aerodynamic Lift can generally be ignored for normal road riding speeds; which means that: $N_f + N_r = W$

$$Friction\ on\ the\ Front\ wheel = \mu_f\, N_f$$

$$Friction\ on\ the\ rear\ wheel = \mu_r\, N_r$$

Note that the Coefficient of Friction acting on the front contact patch μ_f and that acting on the rear contact patch μ_r are likely to have the same value: but they could also differ!

Weight therefore affects 'grip' indirectly, through its Newton's Third Law relationship with the Normal Load.

Note also that 'load' can be transferred from front wheel to back wheel under acceleration and from back wheel to front wheel under braking and this is referred to as the Transferred Load.

Anything affecting the Normal Load on the contact patches will also affect 'grip'; therefore, Transferred Load will affect 'grip'; as such, the front wheel loses 'grip' under Acceleration and gains 'grip' under Braking; whilst the opposite is true for the back wheel.

Dynamic Load on each wheel: (Motorcycle Dynamics P84)

Dynamic Load = Static load + Transferred Load

With a Driving Force of S (Newtons), the Dynamic Load on the front wheel is given by the formula:

$$N_f = mg\frac{b}{p} - S\frac{h}{p}$$

And the Dynamic Load on the rear wheel is given by the formula:

$$N_r = mg\frac{(p-b)}{p} + S\frac{h}{p}$$

The quantities referred to are shown in the diagram to the right; where p is the wheel base of the motorcycle; b is the horizontal distance from the rear contact patch to the Centre of Gravity and h is the height of the Centre of Gravity above the road plane.

84 Motorcycle Dynamics

force S, and drag force F_D are known.

($\Rightarrow$) Equilibrium of horizontal forces: $S - F_D = 0$

($\Uparrow$) Equilibrium of vertical forces: $mg - N_r - N_f = 0$

($\frown$) Equilibrium of moments with respect to the center of gravity:

$$S\,h - N_r b + N_f (p-b) = 0$$

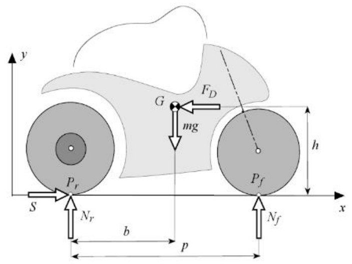


Fig. 3-10 Forces acting on a motorcycle.

The derivations of these formulae are shown at Annex A. They were obtained by considering the various vertical forces and the Moments (I.E. Torques) those forces generate around the Centre of Gravity, as shown in the diagram above.

Transferred Load:

Dynamic Load = Static Load + Transferred Load, and Dynamic Load on the front wheel is:

$N_f = mg\frac{b}{p} - S\frac{h}{p}$ therefore $mg\frac{b}{p}$ is the Static Load and $-S\frac{h}{p}$ is the Transferred Load.

Similarly, the Dynamic Load on the back wheel is: $N_r = mg\frac{(p-b)}{p} + S\frac{h}{p}$ where $mg\frac{(p-b)}{p}$ is the Static Load and $+S\frac{h}{p}$ is the Transferred Load.

In each case the Transferred Load component is $(s\frac{h}{p})$ and this is subtracted from the load on the front wheel and added to the load on the back wheel under dynamic conditions.

I.E. when the bike is accelerating the front wheel loses 'grip' and the back wheel 'gains' grip by an amount equal to $(s\frac{h}{p})$ Newtons.

When braking, the same formulas appear, however, this time they show how the Dynamic Load on the front wheel is increased by the Braking Force (F), whilst that on the rear wheel is reduced by an equal amount; the equations then become:

$$N_f = mg\frac{b}{p} + F\frac{h}{p}$$

$$N_r = mg\frac{p-b}{p} - F\frac{h}{p}$$

This time you can see how load is transferred to the front wheel by an amount $(F\frac{h}{p})$ Newtons when the Braking Force (F) is applied, whilst removing an equal amount of load from the rear wheel.

Motorcycle Dynamics Page 97.

Load Transfer under Acceleration and Braking:

It was stated above that when the motorcycle is moving, the Normal Loads (N_f and N_r) are referred to as the Dynamic Loads.

The Weight of the motorcycle affects the Dynamic Load on each wheel and this in turn influences the size of the Lateral and Longitudinal Forces acting on each tyre contact patch: in other words, Weight affects 'grip', even although Weight itself acts down the way and not laterally or longitudinally.

Lateral Friction Force (F_s) is trying to prevent the tyres from slipping out from under the bike when it is leaning off the vertical and are therefore central to our quest to know whether 'grip' increases or decreases with Lean Angle.

It was said above that 'grip' increases with Roll Angle. (Motorcycle Dynamics Page 52).

Professor Cossalter assigns Camber the Greek letter Phi (φ) and leads to a formula for Lateral

52 *Motorcycle Dynamics*

2.5.5 Lateral force needed for motorcycle equilibrium

Consider a motorcycle in a curve in steady state. The equilibrium of the moments of the forces acting on the center of mass shows that the normalized lateral force necessary to assure the motorcycle's equilibrium is equal to the tangent of the roll angle, as represented in Fig. 2-17.

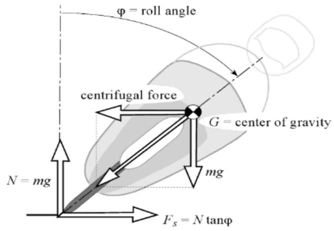
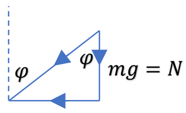


Fig. 2-17 Equilibrium of the motorcycle in a curve.



$$F_s = Centrifugal\ Force$$

$$Tan\varphi = \frac{F_s}{N}$$

$$F_s = NTan\varphi$$

Force (F_s), one of the two Friction Forces which comprise overall 'grip', where $F_s = N\ Tan\varphi$

F_s is the Lateral Force on the tyre;

N is the Normal Load on the tyre;

$Tan\varphi$ is the Tangent of the Camber Angle (I.E. the Lean Angle of the bike.)

This article set out to investigate the relationship between 'grip' and Lean Angle and this formula for Lateral Force (I.E. Lateral Friction, one of the main components of overall 'grip') gives us the answer!

Lateral Force (F_s) increases with $Tan\varphi$, and $Tan\varphi$ increases as the Lean Angle (φ) increases; therefore 'grip' increases with Lean Angle!

At this point you may well be asking the question: '*so why do the wheels slip from under a bike when the Lean Angle is high, if 'grip' increases with Lean Angle?'*

It must be understood that F_s (the Lateral Force) is a statement of the ***theoretical grip needed*** for the bike to be dynamically stable in a bend.

$Tan\varphi\ is\ zero$ when the bike is upright. I.E. when Camber = 0^0 $Tan\ 0^0 = 0$

$Tan\varphi$ is infinity, when Camber is 90^0 I.E. when Camber $=90^0$ $Tan 90^0 = \infty$

On entering a bend, the rider initiates Lean, normally through Positive Steering. The bike continues to increase the Lean Angle until the Moment (another name for Torque) generated by the Weight around the Roll Axis (I.E. the imaginary line joining the front and rear contact patches) trying to make the bike fall into the curve, is equal and opposite to the Moment generated by the Centrifugal Force around the Roll Axis, trying to counter the roll.

At that point, the Weight vector and the Centrifugal Force vector add up, so that their Resultant Force is inclined at the Lean Angle and passes through the Roll Axis.

When this condition is reached, the bike attains dynamic stability in the bend and retains the Lean Angle, providing all other factors remain constant.

However, this 'need' for 'grip' can ***only*** be met by the bike for Lean Angles (I.E. Camber Angles) up to some limiting value, which depends on the tyre structure and the Coefficient of Friction that applies for the tyre / road surface combination.

Hence why the need for grip with a Camber of 90^0 is infinite, because $Tan\ 90^0$ is 'infinity'; but clearly no bike tyre can provide infinite grip and the tyres will lose adhesion on the road surface and slip out sideways from under the bike long before the Camber reaches 90^0; for example, the maximum Camber may be 50^0 or 60^0 for race bikes and less for road bikes.

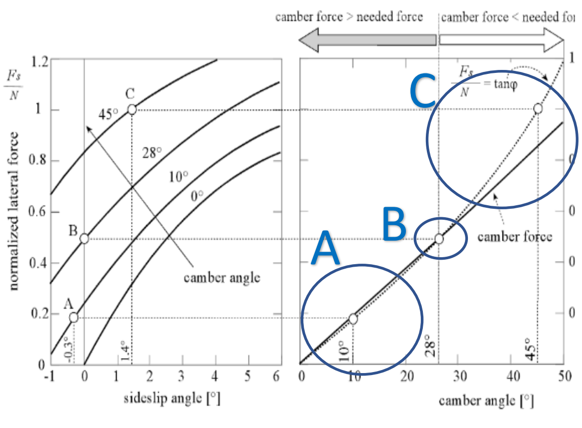


Fig. 2-18 Components of the lateral force generated by camber and slip. Tire A.

Motorcycle Dynamics Page 53 Fig.2.18

Look at the diagram on the right of Fig 2-18 where I have identified three areas within circles, which I have referred to as A, B & C.

The dotted line in this diagram is simply a graph of $Tan\varphi$, which from our $F_s = N\ Tan\varphi$ equation is $\frac{F_s}{N}$ Recall that this is the theoretical 'grip' needed for stability in a bend.

The real Frictional Forces required to meet this need are:

- The Camber Thrust, (or Camber Force as it is referred to in this diagram) caused by the elasticity of the tyre and its resistance to being deformed in the bend. Camber Thrust increases linearly with Roll Angle and is shown in the diagram by the solid black line sloping up from left to right;

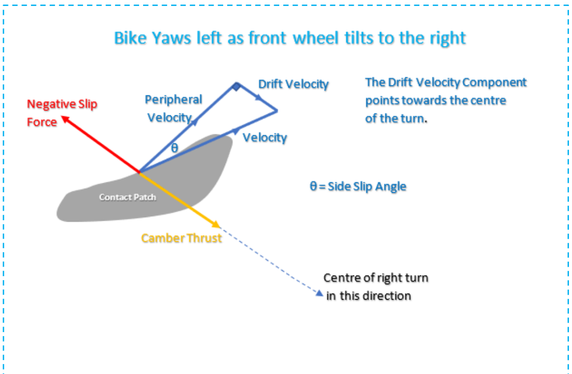
- And the Slip Force, which is the Lateral Friction Force that opposes the tyre slipping out from under the bike.

Look at the area in the diagram shown within Circle A:

Note that the 'needed' Force is less than the Camber Thrust. I.E. the dotted $Tan\varphi$ line ***is below*** the Camber Thrust line. Mathematically this is telling us that for Roll Angles from 0^0 to 28^0, Camber Thrust is too big and needs to be 'detuned' by the Slip Generated Force. It should also be noted that there is nothing 'magic' about this 28^0 Roll Angle: it just happens to be the Roll Angle for ***this tyre***, at which the Needed Force for stability is met entirely by the Camber Thrust: it will be a different Roll Angle for a different tyre!

How can the Slip Force 'detune' the Camber Thrust? The wheel must display a velocity component towards the centre of the turn, and this happens within the initial stages of a turn, which the rider initiates by 'Positive Steering': that is, the rider pushes away on the right handlebar end to go right; or the left handlebar end to go left.

The bike then immediately yaws in the opposite direction to the required turn: a phase of the sequence known as 'out-tracking'. Consider a rider initiating a turn to the right; the Lateral Force on the front tyre contact patch initially points left, as a result of the rider's input torque on the handlebars. The bike yaws to the left and immediately generates Inertia to the right, to oppose the Centripetal Acceleration around the yaw centre of turn to the left. The Inertia force causes the front wheel to start leaning to the right!

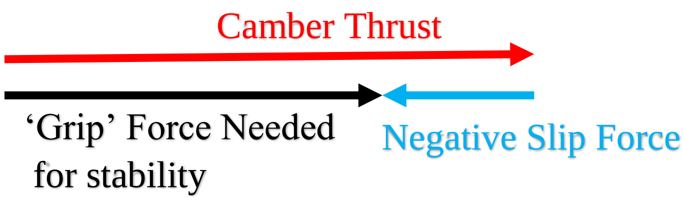


At this point there are two opposing Lateral Forces on the front tyre contact patch; the Lateral Force to the left generated when the rider pushes away on the right handlebar end; and the Camber Thrust to the right, generated by elastic distortion of the tyre carcass.

Over the next few seconds as the front wheel leans more to the right, the Camber Thrust increases with lean angle and effectively reduces the left pointing Lateral Force, which reduces to zero before building up in the opposite direction. In other words, the left facing Lateral Force oppose the Camber Thrust as the front wheel sweeps from left to right to bring the motorcycle round onto the required track through the right bend. If the Camber

Thrust is considered to be a positive force, the Lateral Force at this time must be negative, as it points in the opposite direction: it is the negative Slip Force!

This is shown below using a Force vector diagram:



A 'negative' force is simply one pointing in the opposite direction to another force that has been declared to be positive!

By convention, Slip Generated Force is considered to be positive, when it points in the same direction as the Camber Thrust and it is assumed to be negative when it opposes the Camber Thrust.

Therefore the 'direction' of vectors is important; mathematically the diagram above shows that:

Red + Blue = Black

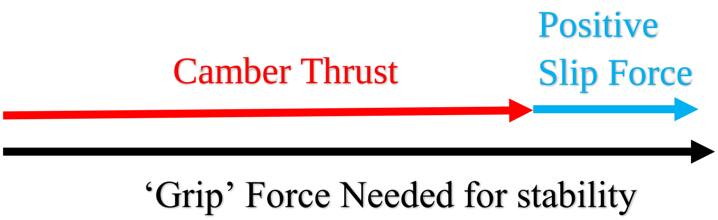
(I.E. vectors are added by placing the 'arrows' tip to tail).

Now look at the area in the diagram shown within Circle B:

If the Roll Angle is equal to 28^0, then <u>for this tyre</u>, the Needed Force is met entirely by the Camber Thrust and the tyre does not slip at all. I.E. Slip Force = 0

Red = Black

Now look at the area in the diagram shown within Circle C:



When the Roll Angle is greater than 28^0 <u>for this tyre</u>, Camber Thrust alone cannot meet the Needed Force for stability. The 'difference' is made up by the tyre slipping away from the centre of the turn, generating a positive Slip Force, which then adds to the Camber Thrust to generate the right amount of 'grip' for stability in the bend.

Red + Blue = Black

'Lateral Grip' is therefore the sum of Camber Trust and Slip Force, both of which act at right angles to the wheel's direction of travel and point towards the centre of the bend.

The area of the graph shown by Circle C, also shows us how the theoretical 'needed grip' increases towards infinity as the Roll Angle increases towards 90^0; however, the Slip Generated Force is limited and the tyre will reach a point at which the tension within the carcass overcomes the tyre's ability to adhere to the road; at this point there will be no adhesion; the entire contact patch area will then be slipping, and the bike will fall over owing to excessive lean and zero adhesion.

2.6 Moments acting between the tire and the road

2.6.1 Self-alignment moment

The distribution of the lateral shear stress generated by the lateral slip of the tire is not symmetric. The resulting force is therefore applied at a point situated at a certain distance from the center of the patch, a center which, in a first approximation, can be assumed to coincide with the theoretical contact point of the rigid toroid with the road plane. The distance a_t is designated the trail of the tire or pneumatic trail. It is clear from Fig. 2-24 that the lateral force generates a moment that tends to rotate the tire in such a way as to diminish the slip angle. For this reason this moment is called the self-aligning moment of the tire.

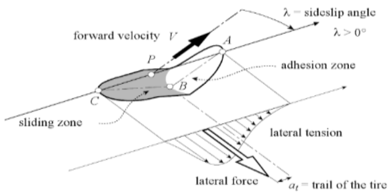


Fig. 2-24 Trail of the tire.

Motorcycle Dynamics Page 59 Fig 2-24.

The diagram above shows how stress and adhesion combat each other across the contact patch. In summary, Lateral Grip (F_S) is made up from the vector sum of two Lateral Forces acting on the tyre contact patches, namely Camber Thrust, which is dependent on the geometry of the tyre; and Lateral Slip, (referred to as the Slip Force) which is a Friction Force generated as the Contact patch slips sideways in the bend.

The leading part of the Contact Patch generates the Slip Force through adhesion to the road surface, but the Lateral Tension in the tyre owing to its elasticity, builds up over the length of the contact patch; and once this Tension exceeds the Force of Adhesion between the tyre and the road surface, the trailing part of the contact patch slips laterally (I.E. sideways away from the centre of the bend).

As the roll increases, so does the Lateral Tension in the tyre and once all the Contact Patch area is slipping, with no adhesion, the tyre has reached its limit of lean and any further increase in Camber will cause the tyre to slip sideways and the bike will fall.

Conclusion:

Increased Lean Angles demand a greater degree of 'grip' and up to a limiting point, the tyres provide this 'grip' through a combination of Camber Thrust and Slip Generated Force: both of which are Lateral Friction Forces. Once the Lean Angle reaches a critical value for that tyre / road surface combination and the contact patches lose adhesion, the tyres slip sideways away from the centre of turn and the bike goes down.

'Grip' also includes a Longitudinal Friction component generated by either a driving force of braking force, depending on the rider's input at the time.

Both the Lateral Friction and Longitudinal Friction components are linked by the need to comply with the limits set for overall friction, by the Friction Ellipse.

George A Cairns B.Sc. Cert. Ed.

Chief Observer Carlisle

West Cumbria Advanced Motorists

(See Annex A below)

Annex A:

To prove the formulae for Normal Load on the front and back wheels:

$$N_f = mg\frac{b}{p} - S\frac{h}{p}$$

$$N_r = mg\frac{p-b}{p} + S\frac{h}{p}$$

Proof:

From the diagram on the right when the bike is in equilibrium opposing Forces and their resultant Moments (I.E. Torques) are equal.

84 **Motorcycle Dynamics**

force S, and drag force F_D are known.

($\Rightarrow$) Equilibrium of horizontal forces: $S - F_D = 0$

($\Uparrow$) Equilibrium of vertical forces: $mg - N_r - N_f = 0$

($\curvearrowleft$) Equilibrium of moments with respect to the center of gravity: $S\,h - N_r b + N_f(p-b) = 0$

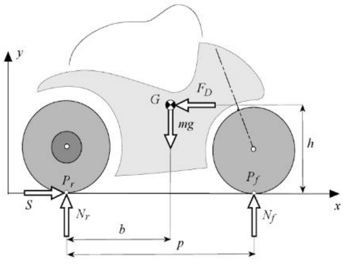


Fig. 3-10 Forces acting on a motorcycle.

$S = F_D$ --------------------1 S is the Driving Force and F_D is the Aerodynamic Drag Force

$mg = N_f + N_r$---------------2 mg is the Weight; N_f & N_r are the Normal loads on front and back wheels

$Sh + N_f(p-b) = N_r b$---3 h is the height of the CG; p is the wheel base and b is the horizontal distance from the rear contact patch to the CG

Equation 3 above is derived by equating the Moments (I.E. Torques) trying to rotate the bike ACW about the Centre of Gravity, with those trying to rotate it CW. E.G. Driving Force (S) times height h to the Centre of Gravity, provides an ACW Moment; as does N_f the Normal Load on the front wheel times the distance $(p-b)$: so we add these two Moments together. For a state of equilibrium to exist, these ACW Moments must equal the CW Moments acting on the bike and there is only one, which is $N_r b$. Note that the Aerodynamic Drag Force F_D cannot exert any Moment about the Centre of Gravity, as it passes through it!

From Equation 2 above:

$$N_r = mg - N_f$$

Substituting for N_r in Equation 3:

$$Sh + N_f(p-b) = (mg - N_f)b$$

$$Sh + N_f p - N_f b = mgb - N_f b$$

$$Sh + N_f p - N_f b + N_f b = mgb$$

$$N_f p = mgb - Sh$$

$$\boldsymbol{N_f = mg\frac{b}{p} - S\frac{h}{p}}$$

Substituting for N_f in Equation 2 above:

$$mg = \left(mg\frac{b}{p} - S\frac{h}{p}\right) + N_r$$

$$N_r = mg - (mg\frac{b}{p} - S\frac{h}{p})$$

$$N_r = mg\left(1 - \frac{b}{p}\right) + S\frac{h}{p}$$

$$\boldsymbol{N_r = mg\frac{p-b}{p} + S\frac{h}{p}}$$