Acceleration remains constant throughout the speed range of your bike: True or False?

This statement is ***False***.

At first sight it would seem logical that your bike has ***constant acceleration*** across the speed range, but this is not the case.

More work must be done to push air aside as speed increases, because Aerodynamic Resistance, also known as Drag, increases rapidly with speed. I.E. if speed doubles Drag increases four-fold, and this is also why fuel consumption rises rapidly as speed increases. With increasing Speed, the engine does more and more work to overcome Drag, leaving less available to accelerate the bike: the ***rate of Acceleration*** therefore decreases as speed increases.

If Acceleration is not constant, is it possible to identify when it is a maximum and to then determine this maximum value? The answer to both questions is 'yes', so let us look at the detail.

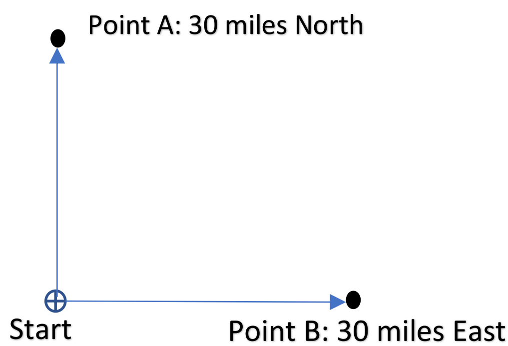
When you get away from talking about concepts and start to explore things in detail, you need to be clear about each quantity; its scientific definition; the units used to measure it and whether it is a Vector quantity (I.E. one with direction, such as Velocity) or a Scalar quantity (I.E. one that does not have a direction element, such as Speed).

Speed is found by dividing the distance travelled by the time taken to travel that distance: to be precise, this is Average Speed.

$$Average\ Speed = \frac{Distance}{Time}$$

The formula for speed is sated in the units used to measure it. We measure speed in 'miles per hour' or 'metres per second': in each case Speed is 'Distance' divided by 'Time'.

Speed is a quantity with no direction. 30mph is 30mph no matter which direction you travel.
In Physics quantities without direction are known as Scalar quantities: Speed is a Scalar quantity.
Velocity is speed ***in a given direction***. E.G. '30mph due North', is a Velocity. '30mph due East' is another Velocity. Quantities with a direction element are referred to as Vector quantities: Velocity is therefore a Vector quantity.
To show the significance of this, look at the diagram to the right. Although Speed and Velocity have the same units, (mph for example) they ***are not*** the same thing. Consider two bikers who start their ride from the same point. One rides due North for an hour at an average speed of 30mph and finishes up at Point A.



The other rides due East for an hour at an average speed of 30mph and finishes up at Point B. Although each had the same speed, they did not finish up in the same place. This is because their Velocities were different.

Now consider Acceleration. When Physicists and Engineers talk about Acceleration, they are referring to the 'Rate of Change of Velocity'.

$$Acceleration = \frac{Velocity}{Time}$$

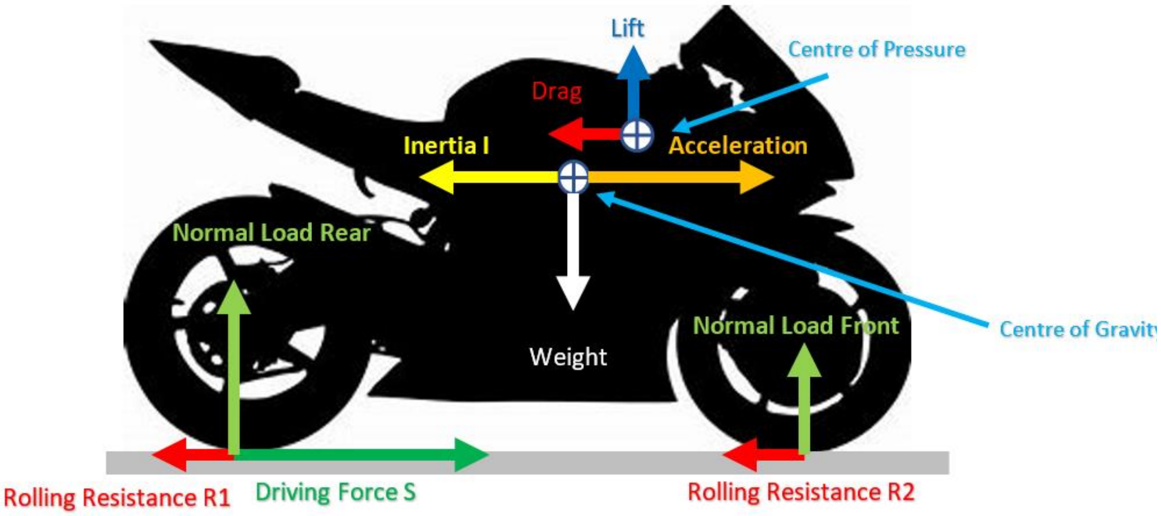
The units of Acceleration are any unit of Velocity divided by any unit of Time. In the Metric System this would be 'metres per second per second' also referred to as 'metres per second squared' written as ms^{-2} and pronounced 'metres seconds to the minus 2'.

Acceleration is also a Vector quantity, as it has direction.

To summarise, Speed is the rate of change of distance; Velocity is the rate of change of distance in a given direction and Acceleration is the rate of change of Velocity.

With these definitions in mind, we can now look at the two questions posed above, namely:

- When is Acceleration at its maximum value?
- What is the value of this maximum Acceleration?



This diagram shows the Vectors acting on a bike. Note that the Centre of Pressure, where the Drag and Lift Forces act, is not the same as the Centre of Gravity.

The Yellow arrow represents the Force generated by the bike's Inertia.

Inertia is that property which resists any change in a body's state of rest or uniform motion. Mass is a measure of Inertia. The more massive a body is, the more Inertia it has and the harder it is to get it moving; or to stop it moving once it has started. ***Inertia opposes Acceleration by generating a Force in the opposite direction to the Acceleration***; a Force that resists attempts to get the bike moving; or once it is moving, to resist any attempt to change its speed; hence why the Yellow arrow points in the opposite direction to the bike's movement when it is accelerating.

The Inertia Generated Force opposing Acceleration is goverened by Newton's Second law of Motion.

$$Inertia\ (I) = Mass(m)\ x\ Acceleration(a) \quad \text{Equation 1}$$

Lift is comparatively small at legal speeds, as is the Rolling Resistance Forces R1 and R2 acting on the rear and front wheels respectively, therefore we can discount all of them from our argument.

If we look at the horizontal Forces acting on the bike and assume the engine is powerful enough to maintain a steady velocity, we can write a formula connecting the Driving Force (S), the Inertia Generated Force (I) and the Drag Force (D):

$$S = I + D \quad \text{Equation 2}$$

In words: this formula tells us that during acceleration, the Driving Force (S) is equal to the sum of the Inertia Generated Force (I) plus the Drag Force (D).

If the Driving Force was greater than the maximum Friction Force available at the rear contact patch, the rear wheel would slip and spin instead of gripping.

Friction therefore limits Acceleration and as such, the Driving Force (S) must be less than or equal to the maximum Friction Force that can be generated at the rear contact patch.

The Maximum Friction Force at the patch is given by the formula:

$$F_{Max} = \mu_s N_r \quad \text{Equation 3}$$

Where μ_s $is\ the\ Coefficient\ of\ Static\ Friction$ and N_r $is\ the\ Normal\ Load\ on\ the\ rear\ patch.$

We can therefore say that: $S \leq \mu_s N_r$ Equation 4

In words: this formula says that the Driving Force (S) must be 'less than or equal to' the maximum Friction Force ($\mu_s N_r$).

At Annex A to this article I have derived a formula for N_r the Normal Load on the rear patch, and it is this:

$$N_r = mg\ \frac{(p-b)}{p} + S\frac{h}{p} \quad \text{Equation 5}$$

Where:

$p\ is\ the\ wheel\ base\ of\ the\ bike$

$b\ is\ the\ distance\ from\ the\ rear\ patch\ to\ the\ point\ directly\ below\ the\ Centre\ of\ Gravity$

$h\ is\ the\ height\ of\ the\ Centre\ of\ Gravity\ above\ the\ road$

$m\ is\ the\ mass\ of\ the\ bike\ and\ rider$

$g\ is\ the\ Acceleration\ of\ Gravity\ (9.8\ metres\ per\ second\ squared)$

$S\ is\ the\ Driving\ Force$

Substituting the expression for N_r in Equation 5 into Equation 4 for S, gives Equation 6 below.

$$S \leq \mu_s \left(mg\,\frac{(p-b)}{p} + S\frac{h}{p} \right)$$

Equation 4 becomes: $$S \leq \mu_s mg\left(\frac{(p-b)}{p}\right) + \mu_s\, S\frac{h}{p}$$ ——— Equation 6

$$S\left(1 - \mu_s\frac{h}{p}\right) \leq \mu_s mg\left(\frac{(p-b)}{p}\right)$$

With a little bit of rearranging
We finish with this:

$$S \leq \frac{\mu_s mg\left(\frac{(p-b)}{p}\right)}{\left(1 - \mu_s\frac{h}{p}\right)}$$

Using Equation 2, we can say: $$(I + D) \leq \frac{\mu_s mg\left(\frac{(p-b)}{p}\right)}{\left(1 - \mu_s\frac{h}{p}\right)}$$

$$I \leq \left(\frac{\mu_s mg\left(\frac{(p-b)}{p}\right)}{\left(1 - \mu_s\frac{h}{p}\right)}\right) - D$$

Using Equation 1, we can say: $$m \times a_{max} \leq \left(\frac{\mu_s mg\,\frac{(p-b)}{p}}{1 - \mu_s\frac{h}{p}}\right) - D$$

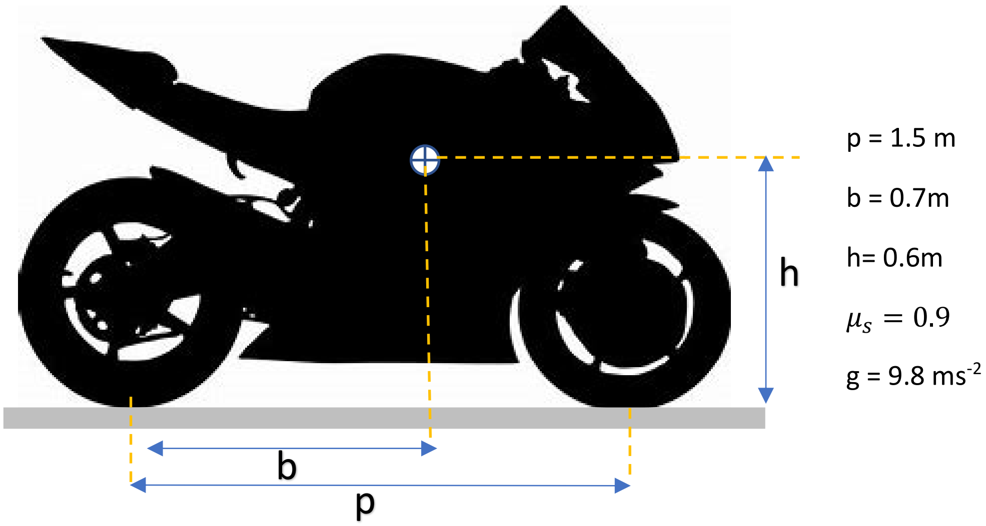
Dividing every term by mass (m): $$a_{max} \leq \left(\frac{\mu_s g\,\frac{(p-b)}{p}}{1 - \mu_s\frac{h}{p}}\right) - \frac{D}{m}$$ ——— Equation 7

Equation 7 tells us all we need to know about Maximum Acceleration (a_{max}).

In words: this formula says that when the Drag Force (D) is zero, the term $\frac{D}{m}$ is also zero, and when this is the case Acceleration will be at its maximum value (a_{max}). It also says that (a_{max}) must be less than or equal to the value within the big brackets.

__Maximum Acceleration occurs when Drag is zero__ and that is when the bike is starting from low speed. Thereafter, increasing Speed increases Drag and a greater proportion of the Driving Force goes into overcoming this Drag, which causes the rate of Acceleration to reduces as Speed increases.

Irrespective of how powerful the engine is, Acceleration is limited by Traction at the rear contact patch. Once the Driving Force exceeds available Traction, the rear wheel slips and Acceleration does not increase. The value of the Maximum Acceleration is therefore given by Equation 7, so let us put some typical values for a sports bike into the equation and work out the maximum Acceleration!



When moving off, we can assume that Drag = 0. Equation 7 then tells us:

$$a_{max} \le \left(\frac{\mu_s g \frac{(p-b)}{p}}{1 - \mu_s \frac{h}{p}} \right) - 0$$

Putting in the parameters for this bike:

$$a_{max} \le \left(\frac{0.9 \; x \; 9.8 \frac{(1.5-0.7)}{1.5}}{1 - 0.9 \; x \; \frac{0.6}{1.5}} \right) metres\ per\ second\ squared$$

$$a_{max} \le 7.34\ metres\ per\ second\ squared$$

That may not ***seem*** very much, but it equates to a speed of 163.3mph after 10 seconds of acceleration, assuming the acceleration remains constant, which of course we have just argued that it does no due to increasing Drag with speed. What we can say is that in the absence of Drag, this Acceleration would result in a speed of 163.3mph, 10 seconds after moving away from stationary.

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Diagram from 'Motorcycle Dynamics' an excellent publication by Professor Vittore Cossalter.

Annex A:

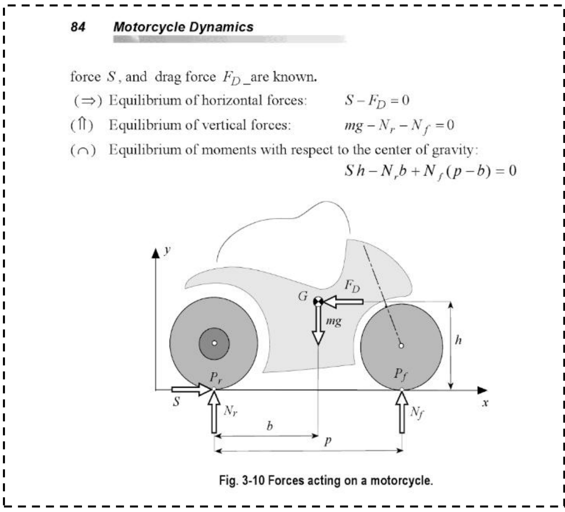
To prove the formulae for Normal Load on the front and back wheels:

$$N_f = mg\frac{b}{p} - S\frac{h}{p}$$

$$N_r = mg\frac{p-b}{p} + S\frac{h}{p}$$

Proof:

From the diagram on the right when the bike is in equilibrium opposing Forces and their resultant Moments (I.E. Torques) are equal.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force S, and drag force F_D are known.

($\Rightarrow$) Equilibrium of horizontal forces: $S - F_D = 0$

($\Uparrow$) Equilibrium of vertical forces: $mg - N_r - N_f = 0$

($\curvearrowleft$) Equilibrium of moments with respect to the center of gravity: $S\,h - N_r b + N_f(p-b) = 0$

Fig. 3-10 Forces acting on a motorcycle.

$S = F_D$ --------------------1 S is the Driving Force and F_D is the Aerodynamic Drag Force

$mg = N_f + N_r$---------------2 mg is the Weight; N_f & N_r are the Normal loads on front and back wheels

$Sh + N_f(p-b) = N_r b$---3 h is the height of the CG; p is the wheelbase and b is the horizontal distance from the rear contact patch to the CG

Equation 3 above is derived by equating the Moments (I.E. Torques) trying to rotate the bike ACW about the Centre of Gravity, with those trying to rotate it CW. E.G. Driving Force (S) times height h to the Centre of Gravity, provides an ACW Moment; as does N_f the Normal Load on the front wheel times the distance $(p-b)$: so we add these two Moments together. For a state of equilibrium to exist, these ACW Moments must equal the CW Moments acting on the bike and there is only one, which is $N_r b$. Note that the Aerodynamic Drag Force F_D cannot exert any Moment about the Centre of Gravity, as it passes through it!

From Equation 2 above:

$$N_r = mg - N_f$$

Substituting for N_r in Equation 3:

$$Sh + N_f(p-b) = (mg - N_f)b$$

$$Sh + N_f p - N_f b = mgb - N_f b$$

$$Sh + N_f p - N_f b + N_f b = mgb$$

$$N_f p = mgb - Sh$$

$$\boldsymbol{N_f = mg\frac{b}{p} - S\frac{h}{p}}$$

Substituting for N_f in Equation 2 above:

$$mg = \left(mg\frac{b}{p} - S\frac{h}{p}\right) + N_r$$

$$N_r = mg - (mg\frac{b}{p} - S\frac{h}{p})$$

$$N_r = mg\left(1 - \frac{b}{p}\right) + S\frac{h}{p}$$

$$\boldsymbol{N_r = mg\,\frac{p-b}{p} + S\frac{h}{p}}$$