DOES THE WEIGHT OF A BIKE AFFECT THE ANGLE AT WHICH IT LEANS IN A BEND?

This seems like a reasonable question: if you take a bend at a certain speed when two up, does the bike lean more than it does when one up?

Well the answer is NO.

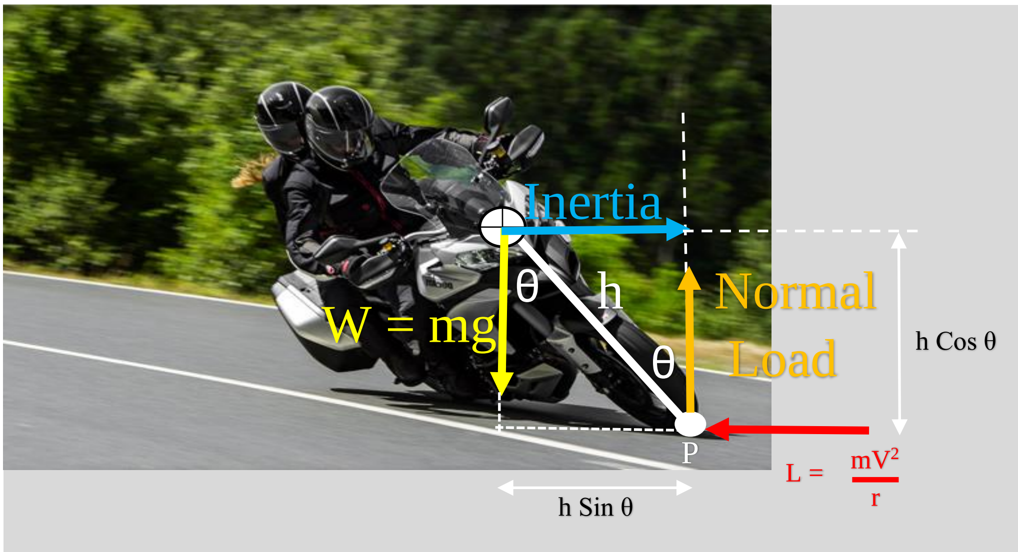
What does affect the Lean Angle is the ***speed*** of the bike and the ***radius*** of its track through the bend.

The weight of the bike, whether it's a big bike or a small bike; whether it's one up or two up makes no difference to the lean angle!

How do we know this is the case?

By analysing the physics of a bike leaning over in a bend, we can derive a formula that allows us to calculate the 'lean angle'. If the formula ***does not*** contain a reference to the mass of the bike and rider; then the mass will not affect the lean angle!

To show that this is the case, let's look at the bike shown above and add in a few forces and measurements:



In the diagram above:

- The Weight (W) of the bike and rider is their combined Mass (m) multiplied by the acceleration of Gravity (g): W=mg;

- The height of the Centre of Gravity above the contact patch (P) is h, when the bike is sitting upright;

- The Lean Angle of the bike off the vertical (also known as the Camber Angle or simply Camber) is shown by the Greek letter Theta (θ);

- When a motorcycle negotiates a bend, it experiences a force towards the centre of the bend known as the Corner Force. In Physics it is referred to as the Centripetal Force, which literally means the 'centre facing force' as it points towards the centre of the bend. The Corner Force is a demand which is met by the combined effects of Camber Thrust and Slip Force. Camber Thrust acts like a spring resisting the deformation of the tyre as the weight of the bike pushes it down onto the road surface when leaning into a bend. The Slip Force is the frictional resistance opposing the tyre as it slides across the road surface towards the outside of the bend. Together they add to provide the Lateral Force, which we have shown in the diagram as $L = \frac{mV^2}{r}$ where V is the Velocity of the bike and (r) is the radius of its track through the bend. It will be shown later that $L = N\ Tan\theta$. This tells us that the Lateral Force (or Grip) increases with the lean angle: but only up to a point! Beyond a critical lean angle, which varies from bike to bike, the tyres let go and the bike falls over in the bend.

- Inertia is a force that ***opposes*** acceleration. Both Centripetal Acceleration and the associated Corner Force act in towards the centre of the turn, therefore the Inertia Force acts in the opposite direction, pointing away from the centre of turn. Inertia is the Newton's Third Law 'equal and opposite force' to the Corner Force demand and as such it is also equal to $\frac{mV^2}{r}$
Note: Inertia ***IS NOT*** Centrifugal Force. Centrifugal Force is a Fake Force, or a Pseudo Force introduced when viewing the motion of the bike from a 'Non-Inertial Frame of Reference'. The distinction between Inertial and Non-Inertial Frames of Reference and why there is a need to introduce pseudo forces is a technical matter discussed in another Blog on this website entitled: 'Centrifugal Force Fact or Fiction'

- The off-set distance from the Centre of Gravity to the Contact Point (P) is h Sinθ.
I.E. when the bike leans off the vertical, the Weight (W) no longer acts vertical down through the 'Roll Axis' (I.E. the imaginary line that joins the front and back tyre Contact Patches).
As shown in the diagram above, the offset distance from W to the front Contact Point (P) is given is h Sinθ, where 'h' is the hypotenuse of the triangle containing the Lean Angle θ.

- From the same triangle, the height of the Centre of Gravity above the Contact Point (P) is h Cosθ.

The rider initiates 'lean' at the appropriate point on entering the bend by applying Positive Steering; and leaning the bike into the bend.

There is then a balancing act between the various Torques (also referred to as 'Moments') and forces acting on the bike and when they equal out, the bike achieves 'dynamic balance'.

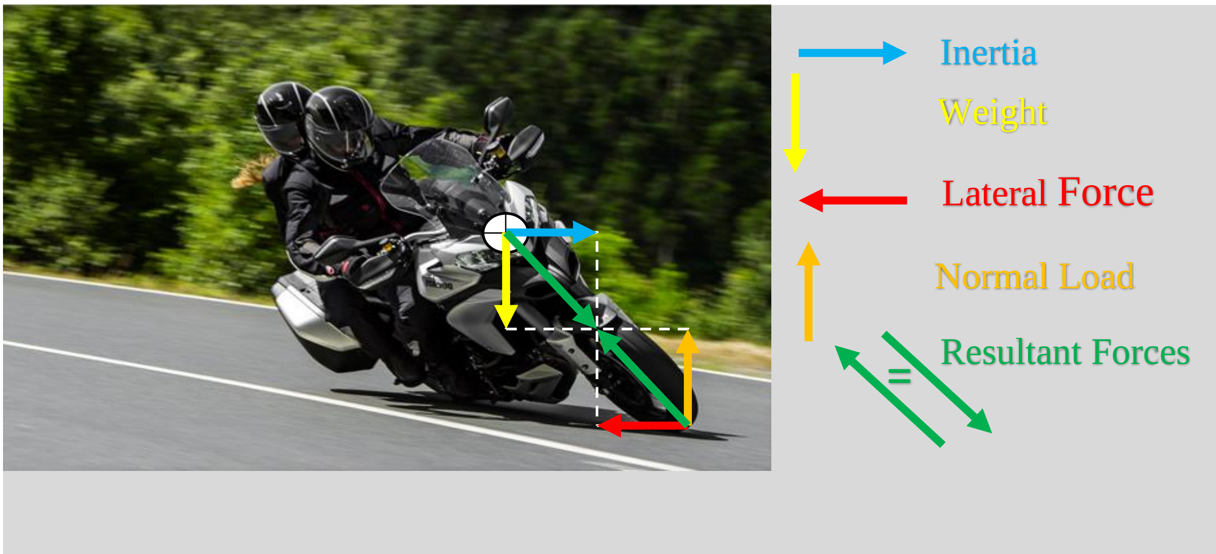
I.E. The Torques trying to lean the bike in are:

- Weight generated Torque;
- The Twisting Moment generated by the profile of a motorcycle tyre (which gives rise to the Rolling Cone Effect);
- Normal Load generated Torque;

Opposing these are the Torques trying to make the bike sit up, they are:

- Inertia generated Torque;
- Torque generated by the Lateral Force on the Contact Patch;
- Torque generated by the Gyroscopic effect of the front wheel.

When Dynamic Balance is achieved, the resultant of the Weight and Inertia Forces is ***equal and opposite*** to the resultant of the Normal Load and Lateral Forces, as shown in the diagram below:



The Weight now exerts a Torque (**T_1**) about the Contact Point. Torque is the 'turning effect of a force' and the size of the Torque is found by multiplying the Force by the offset distance from the point of application of the Force, to the point about which it tries to rotate the bike. The size of **T_1** is therefore the product of the Weight of the bike and rider and the offset distance to the Roll Axis:

$$T_1 = mghSin\theta$$

Note: The ***direction*** in which Torque **T_1** acts for our bike leaning to the right, is directly out the ***front*** of the bike, in the direction of travel; another of those quirky Physics concepts due to the fact that Torque and its component parts (Force and Distance) are all Vector quantities, with defined directions.

However, this does not detract from our goal to see if Weight affects Lean Angle, because it is the ***size*** of the Torque we are interested in and the fact that it is trying to rotate the bike ***counter clockwise,*** when looking at the bike coming towards you.

If your side stand accidentally flips up when the bike is at rest and leaning over, you will have a costly demonstration of the effect of Torque **T_1**

Inertia also exerts a Torque (**T_2**) about the Contact Point (P), where: $T_2 = \frac{\mathrm{mV^2}}{\mathrm{r}}\ hCos\theta$

For a bike leaning to the right, the direction in which Torque $\mathbf{T_2}$ acts, is directly out the ***back*** of the bike, away from the direction of travel and opposite to the direction in which Torque $\mathbf{T_1}$ acts: in other words, $\mathbf{T_1}$ and $\mathbf{T_2}$ ***oppose*** each other.

Again, it's the size of the Torque we are interested in and the fact that it is trying to rotate the bike ***clockwise***, when looking at the bike coming towards you.

As the lean angle θ ***increases***, Sin θ ***increases*** but Cos θ ***decreases***.

The significance of this is that, Torque $T_1\ which\ equals\ mghSin\theta$ ***increases*** as the lean angle θ ***increases*** and Torque $T_2\ which\ equals\ \frac{\mathrm{mV}^2}{\mathrm{r}}\ hCos\theta$ ***decreases*** as the lean angle θ ***increases***.

At some point they become ***equal;*** the bike stops leaning and achieves Dynamic Balance in the bend.

To determine the lean angle (θ) at the point of Dynamic Balance, we simply equate the condition where $T_1 = T_2$ as shown below:

$$T_1 = T_2$$

$$mghSin\theta = \frac{mv^2}{r}\ h\ Cos\theta$$

The 'm' and 'h' terms cancel each other out

Rearranging the equation gives this

$$\frac{Sin\theta}{Cos\theta} = \frac{V^2}{rg}$$

$$\frac{Sin\theta}{Cos\theta} = Tan\theta$$

$$Tan\theta = \frac{V^2}{rg}$$

$$\theta = Tan^{-1}\frac{V^2}{rg}$$

The Lean Angle is the angle whose Tangent is the square of the bike's Velocity; divided by the radius of the bike's track through the bend multiplied by the acceleration of Gravity.

This part of the equation also shows that:

$$mg\ \frac{Sin\theta}{Cos\theta} = \frac{mV^2}{r}$$

$$mg\ Tan\theta = L$$

$$L = N\ Tan\theta$$

I.E. The Lateral Force increases with Lean Angle (but only up to a point!)

You can see that the equation for Lean Angle does not contain the Mass (**m**) of the bike and rider, which allows us to confidently say that:

THE MASS OF THE BIKE AND RIDER DOES NOT AFFECT THE LEAN ANGLE

George A Cairns

CWCAM Chief Observer